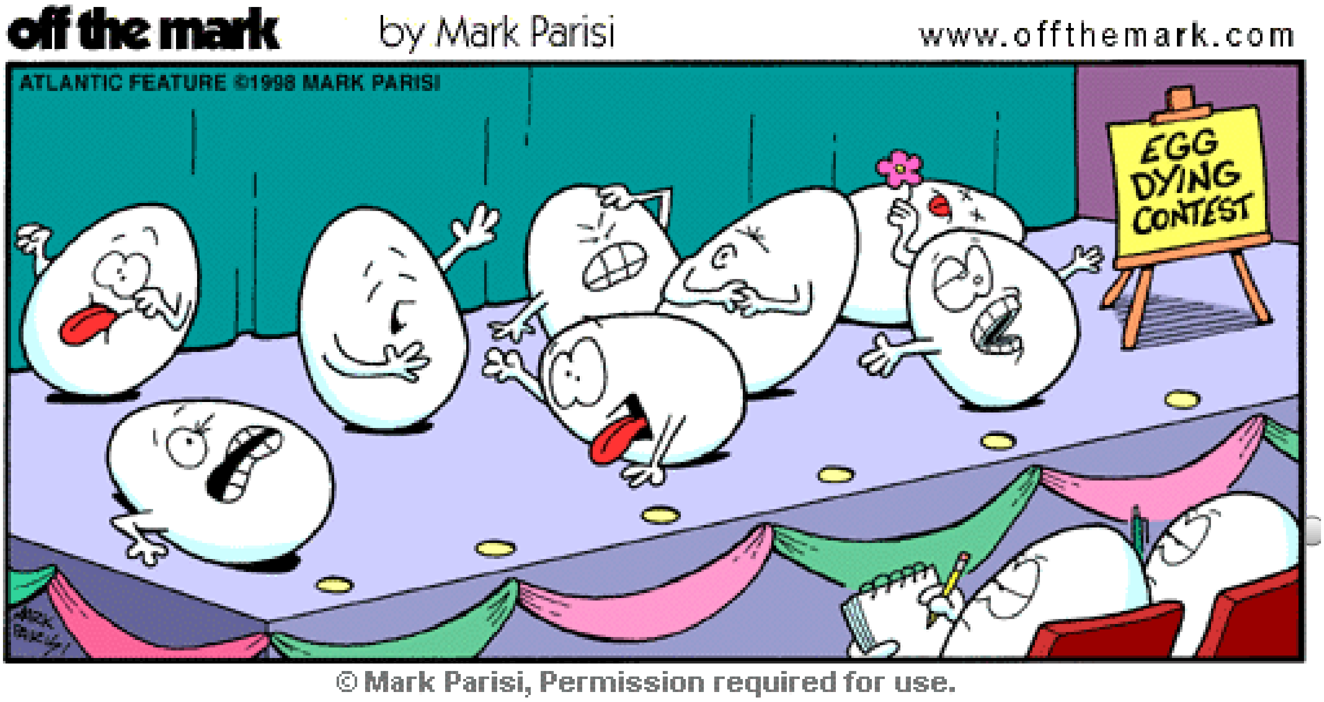
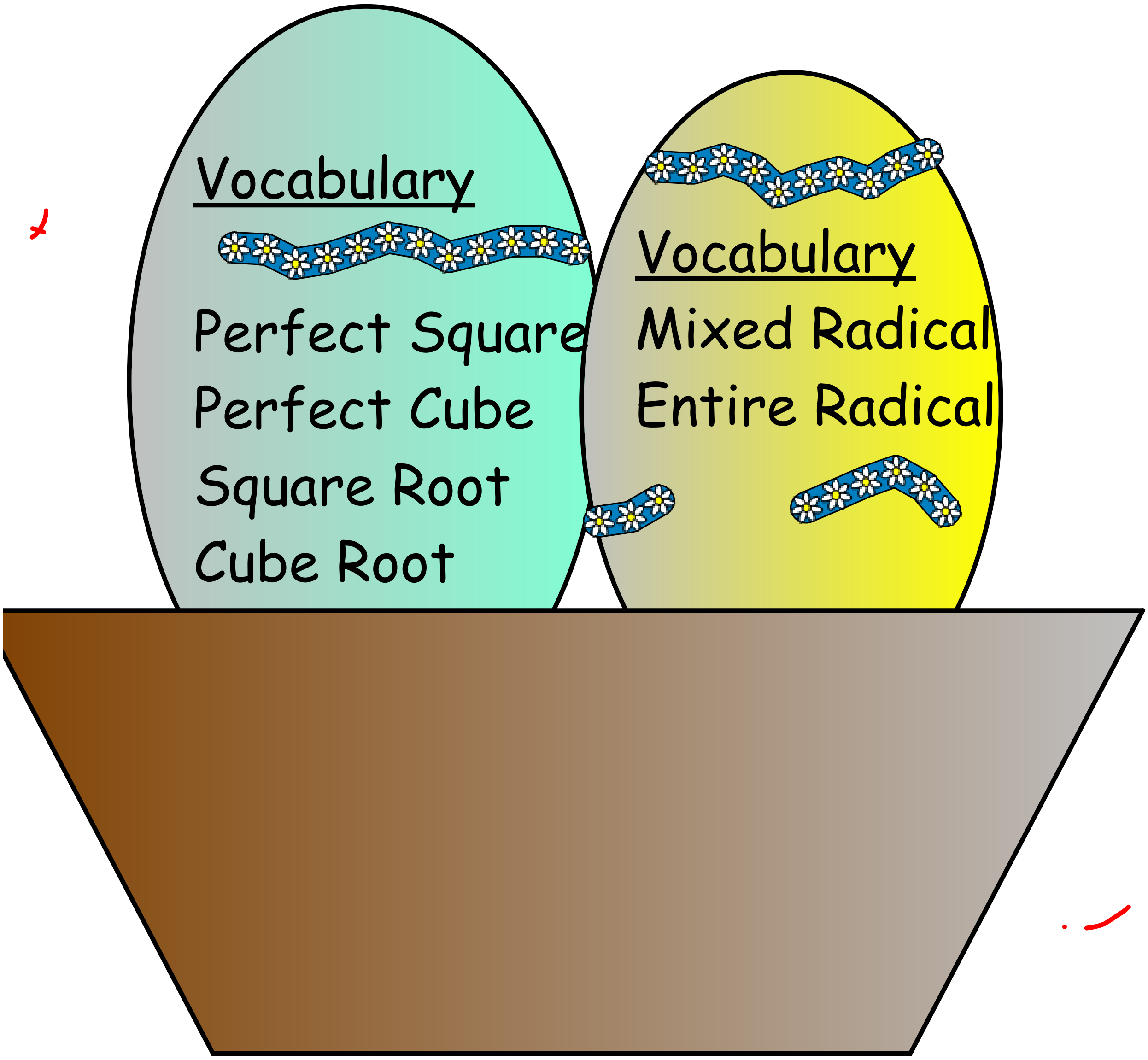
$\sqrt[2]{16}$ $\sqrt[3]{16}$

Lesson -Mixed and Entire Radicals INDEX 2 -




Vocabulary
Perfect Square
Perfect Cube
Square Root
Cube Root
Vocabulary
Mixed Radical
Entire Radical

Number x	Perfect Square x^2	Perfect Cube x^3	Perfect Cube (-) $(-x)^3$
1	1	1	-1
2	4	8	-8
3	9	27	-27
4	16	64	-64
5	25	125	-125
6	36	216	-216
7	49	343	-343
8	64	512	-512

-3 9

-5 25

$2\sqrt{4}$ $2\sqrt{4} = \sqrt{64}$ $\sqrt[3]{32}$

$6\sqrt{5}$ $\frac{3}{2} = 1\frac{1}{2}$ $\sqrt[4]{32}$

$5\sqrt[8]{11}$ improper mixed number $\sqrt{30}$

Mixed Radicals

$4\sqrt[5]{16}$ $\frac{7}{4} = 1\frac{3}{4}$ Entire Radicals

$3\sqrt{7}$

Arrange from least to greatest:

$\sqrt{17}$ ~4.1

$\sqrt{9}$ = 3

$\sqrt{12}$ ~3.5

$\sqrt{3}$ ~1.7

Ex 1

a) Write as an ENTIRE RADICAL.

$6\sqrt{5}$ $2\sqrt{4}$ $7\sqrt{3}$ $4\sqrt{3}$

skip 2 lines

$\sqrt{36}\cdot\sqrt{5}$ | $\sqrt{4}\sqrt{4}$ | $\sqrt{49}\sqrt{3}$ | $\sqrt{16}\sqrt{3}$

$\sqrt{180}$ (13.4) entire | $\sqrt{16}$ (1) (4) | $\sqrt{147}$ ~~12.1~~ | $\sqrt{48}$ (2)

b) Order from least to greatest.

$2\sqrt{4}, 4\sqrt{3}, 7\sqrt{3}, 6\sqrt{5}$

Arrange from least to GREATEST.

$6\sqrt{5}$ $2\sqrt{4}$ $7\sqrt{3}$ $4\sqrt{3}$

4.3 Mixed and Entire Radicals

LESSON FOCUS Express an entire radical as a mixed radical, and vice versa.

Make Connections

We can name the fraction $\frac{3}{12}$ in many different ways:

$\frac{1}{4}$ $\frac{5}{20}$ $\frac{30}{120}$ $\frac{100}{400}$

How do you show that each fraction is equivalent to $\frac{3}{12}$?

Why is $\frac{1}{4}$ the simplest form of $\frac{3}{12}$?

$4\sqrt{3}$ mixed radical

$\sqrt{16}\sqrt{3}$

$\sqrt{48}$

↓

entire

$\underbrace{\sqrt{16}}\sqrt{3}$

$4\sqrt{3}$

Copy

STEPS **for writing a Mixed Radical**

1. Find BIGGEST Perfect Square that is a factor of the radicand. (see link for def'n) $\sqrt{\text{radicand}}$

2. ($\sqrt{}$Perfect Square) (other factor)

3. (Simplify 1st bracket)(other factor)

Example 1, page 215

b) $\sqrt[3]{144}$

Simplify each radical.

SOLUTION

a) $\sqrt{125}$

$= \sqrt{25}\sqrt{5}$

$5\sqrt{5}$

CHECK YOUR UNDERSTANDING

4.3 Mixed and Entire Radicals

b) $\sqrt{63}$

$\sqrt{9}\sqrt{7}$

$3\sqrt{7}$

c) $\sqrt{32}$

$\sqrt{16}\sqrt{2}$

$4\sqrt{2}$

$\sqrt{32}$

$\sqrt{4}\sqrt{8}$

$2\sqrt{8}$

$2\sqrt{8}$

$2\sqrt{4}\sqrt{2}$

$2 \cdot 2\sqrt{2}$

$4\sqrt{2}$

Try This one

$$\sqrt{200}$$

$\sqrt{100}\sqrt{2}$

$10\sqrt{2}$

$\sqrt{25}\sqrt{8}$

$5\sqrt{8}$

$5\cdot\sqrt{4}\sqrt{2}$

$5\cdot 2\sqrt{2}$

$10\sqrt{2}$

$\sqrt{4}\sqrt{50}$

$2\sqrt{50}$

$2\sqrt{25}\sqrt{2}$

$2\cdot 5\sqrt{2}$

$10\sqrt{2}$

$\sqrt{4}\sqrt{50}$

$2\sqrt{50}$

$2\sqrt{25}\sqrt{2}$

$2\cdot 5\sqrt{2}$

$10\sqrt{2}$

Some numbers, such as 200, have more than one perfect square factor.
The factors of 200 are: 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200
Since 4, 25, and 100 are perfect squares, we can simplify $\sqrt{200}$ in these ways.

? ? ?

4.3 Mixed and Entire Radicals

$\sqrt{12}$

$\sqrt{4}\sqrt{3}$

$2\sqrt{3}$

$\sqrt{30}$

$\sqrt{30}$ ✓

a) $\sqrt{30}$ b) $\sqrt{64}$ c) $3\sqrt{4}$ d) $2\sqrt{28}$

End of lesson

Please do the worksheet -both sides

VOCABULARY-Recap

Mixed Radical
Entire Radical

Radicals of the form $\sqrt[n]{x}$ such as $\sqrt{80}$, $\sqrt[3]{144}$, and $\sqrt[4]{162}$ are **entire radicals.**

Radicals of the form $a\sqrt[n]{x}$ such as $4\sqrt{5}$, $2\sqrt[3]{18}$, and $3\sqrt[4]{2}$ are **mixed radicals.** Entire radicals were rewritten as mixed radicals in *Examples 1* and *2*.

$\sqrt[n]{x}$ - Entire Radical

$a\sqrt[n]{x}$ - Mixed Radical

Coefficiant > 1.

Arrange from least to greatest:

$7\sqrt{2}$, $\sqrt{100}$, $5\sqrt{3}$, $4\sqrt{6}$

Check Worksheet

Lesson - Mixed and Entire Radicals with Index higher than 2

In Class Assignment -Early Check IN

Lesson - PART 2 -Mixed and Entire Radicals

2

2

$$6\sqrt{5}$$

$$2\sqrt{4}$$

Mixed Radicals

4

$$\sqrt{30}$$

Entire Radicals

$$\sqrt[4]{32}$$

Perfect Cubes

Perfect Fourths

Perfect Fifths

For example, the factors of 24 are: 1, 2, 3, 4, 6, 8, 12, and 24.

- We can simplify $\sqrt{24}$ because 24 has a perfect square factor of 4. Rewrite 24 as the product of two factors, one of which is 4.

  ?

- Similarly, we can simplify $\sqrt[3]{24}$ because 24 has a perfect cube factor of 8. Rewrite 24 as the product of two factors, one of which is 8.

  ?

- However, we cannot simplify $\sqrt[4]{24}$ because 24 has no factors (other than 1) that can be written as a fourth power.

?

4.3 Mixed and Entire Radicals

How can we edit these steps for other roots ? (i.e. $\sqrt[3]{\ }$ and $\sqrt[4]{\ }$)

STEPS **for writing a Mixed Radical**

1. Find BIGGEST Perfect Square that is a factor of the radicand. (see link for def'n)

2. ($\sqrt{\text{Perfect Square}}$) (other factor)

3. (Simplify 1st bracket)(other factor)

Write each radical in simplest form, if possible.

Ex 2

a) $\sqrt[3]{40}$

c) $\sqrt[4]{32}$

SOLUTION

4.3 Mixed and Entire Radicals

Write each radical in simplest form:

$\sqrt[3]{81}$ $\sqrt[3]{128}$ $\sqrt[4]{512}$

CHECK YOUR UNDERSTANDING

2. Write each radical in simplest form, if possible.

a) $\sqrt{30}$ b) $\sqrt[3]{32}$ c) $\sqrt[4]{48}$

4.3 Mixed and Entire Radicals

Example 3 Writing Mixed Radicals as Entire Radicals

Write each mixed radical as an entire radical.

b) $3\sqrt[3]{2}$ c) $2\sqrt[5]{2}$

SOLUTION

4.3 Mixed and Entire Radicals

CHECK YOUR UNDERSTANDING

3. Write each mixed radical as an entire radical.

a) $7\sqrt{3}$ b) $2\sqrt[3]{4}$ c) $2\sqrt[5]{3}$

4.3 Mixed and Entire Radicals

End of lesson

Textbook -Page 218-219
11acegi,12acegi,17,18, 21, 22

Extra Practice (optional) -page 221

Give out assignments

1 Can this number be simplified?

$$\sqrt{10}$$

A Yes

B No

(For another question, go to the next page.)

2 Can this number be simplified?

A Yes

B No

$$\sqrt{27}$$

Example 1 Simplifying Radicals Using Prime Factorization

Simplify each radical.

a) $\sqrt{80}$ b) $\sqrt[3]{144}$ c) $\sqrt[4]{162}$

SOLUTION

Write each radical as a product of prime factors, then simplify.

$$
\begin{aligned}
\text{a) } \sqrt{80} &= \sqrt{8 \cdot 10} \\
&= \sqrt{2 \cdot 2 \cdot 2 \cdot 5 \cdot 2} \\
&= \sqrt{(2 \cdot 2) \cdot (2 \cdot 2) \cdot 5} \\
&= \sqrt{2 \cdot 2} \cdot \sqrt{2 \cdot 2} \cdot \sqrt{5} \\
&= 2 \cdot 2 \cdot \sqrt{5} \\
&= 4\sqrt{5}
\end{aligned}
$$

Since $\sqrt{80}$ is a square root, look for factors that appear twice.

(Solution continues.)

Example 1 Simplifying Radicals Using Prime Factorization

b) $\sqrt[3]{144} = \sqrt[3]{12 \cdot 12}$

$= \sqrt[3]{2 \cdot 2 \cdot 3 \cdot 2 \cdot 2 \cdot 3}$

$= \sqrt[3]{(2 \cdot 2 \cdot 2) \cdot 2 \cdot 3 \cdot 3}$

$= \sqrt[3]{2 \cdot 2 \cdot 2} \cdot \sqrt[3]{2 \cdot 3 \cdot 3}$

$= 2 \cdot \sqrt[3]{2 \cdot 3 \cdot 3}$

$= 2\sqrt[3]{18}$

Since $\sqrt[3]{144}$ is a cube root, look for factors that appear 3 times.

c) $\sqrt[4]{162} = \sqrt[4]{81 \cdot 2}$

$= \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 2}$

$= \sqrt[4]{(3 \cdot 3 \cdot 3 \cdot 3) \cdot 2}$

$= \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3} \cdot \sqrt[4]{2}$

$= 3\sqrt[4]{2}$

Since $\sqrt[4]{162}$ is a fourth root, look for factors that appear 4 times.

CHECK YOUR UNDERSTANDING

4.3 Mixed and Entire Radicals

Example 2 Writing Radicals in Simplest Form

Write each radical in simplest form, if possible.

a) $\sqrt[3]{40}$ b) $\sqrt{26}$ c) $\sqrt[4]{32}$

SOLUTION

Look for perfect *n*th factors, where *n* is the index of the radical.

a) The factors of 40 are: 1, 2, 4, 5, 8, 10, 20, 40
The greatest perfect cube is $8 = 2 \cdot 2 \cdot 2$,
so write 40 as $8 \cdot 5$.

$$\begin{aligned}\sqrt[3]{40} &= \sqrt[3]{8 \cdot 5}\\ &= \sqrt[3]{8} \cdot \sqrt[3]{5}\\ &= 2 \cdot \sqrt[3]{5}\\ &= 2\sqrt[3]{5}\end{aligned}$$

b) The factors of 26 are: 1, 2, 13, 26
There are no perfect square factors other than 1.
So, $\sqrt{26}$ cannot be simplified.

(Solution continues.)

Example 2 Writing Radicals in Simplest Form

c) The factors of 32 are: 1, 2, 4, 8, 16, 32
The greatest perfect fourth power is $16 = 2 \cdot 2 \cdot 2 \cdot 2$,
so write 32 as $16 \cdot 2$.

$$\sqrt[4]{32} = \sqrt[4]{16 \cdot 2}$$
$$= \sqrt[4]{16} \cdot \sqrt[4]{2}$$
$$= 2 \cdot \sqrt[4]{2}$$
$$= 2\sqrt[4]{2}$$

CHECK YOUR UNDERSTANDING

4.3 Mixed and Entire Radicals

Example 3 Writing Mixed Radicals as Entire Radicals

Write each mixed radical as an entire radical.

a) $4\sqrt{3}$ b) $3\sqrt[3]{2}$ c) $2\sqrt[5]{2}$

SOLUTION

a) Write 4 as: $\sqrt{4 \cdot 4} = \sqrt{16}$

$$4\sqrt{3} = \sqrt{16} \cdot \sqrt{3}$$

Use the Multiplication Property of Radicals.

$$= \sqrt{16 \cdot 3}$$

$$= \sqrt{48}$$

b) Write 3 as: $\sqrt[3]{3 \cdot 3 \cdot 3} = \sqrt[3]{27}$

$$3\sqrt[3]{2} = \sqrt[3]{27} \cdot \sqrt[3]{2}$$

$$= \sqrt[3]{27 \cdot 2}$$

$$= \sqrt[3]{54}$$

(Solution continues.)

Example 3 Writing Mixed Radicals as Entire Radicals

c) Write 2 as: $\sqrt[5]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \sqrt[5]{32}$

$$2\sqrt[5]{2} = \sqrt[5]{32} \cdot \sqrt[5]{2}$$
$$= \sqrt[5]{32 \cdot 2}$$
$$= \sqrt[5]{64}$$

CHECK YOUR UNDERSTANDING

4.3 Mixed and Entire Radicals

CHECK YOUR UNDERSTANDING

1. Simplify each radical.

   a) $\sqrt{63}$

   b) $\sqrt[3]{108}$

   c) $\sqrt[4]{128}$

4.3 Mixed and Entire Radicals